Integrales de potencias trigonométricas

a) Calcule las integral que se indican.

1. $\int \sin^3(x)dx.$
2. $\int \cot^2\left(\frac{x}{2}\right)dx.$
3. $\int \csc^2(\sqrt{2}x)dx.$
4. $\int \cos^{-2}(2x+1)dx.$
5. $\int \sec^{-4}(\sqrt{3}x+1)dx.$
6. $\int \sin^2(x)\cos^4(x)dx.$
7. $\int \sec(x)\tan^3(x)dx.$
8. $\int \sec^2(2x)\tan^3(2x)dx.$
9. $\int \csc^{-1}(2x)\sec^{-3}(2x)dx.$
10. $\int \frac{\tan\left(\frac{x}{2}\right)}{\cos^3\left(\frac{x}{2}\right)}dx.$
11. $\int \frac{\csc^2(1-x)}{\tan(1-x)}dx.$
12. $\int \frac{\cos^5(x)}{\sin^{\frac{2}{3}}(x)}dx.$
13. $\int (\sec(x)+\cos(x))^2dx.$
14. $\int \sec^2(x)\sin^2(x)dx.$
15. $\int e^x\sin^2(e^x)dx.$
16. $\int \frac{\sec(\theta)\cos(\theta)\tan(\theta)}{\sec^3(\theta)\sin(\theta)}d\theta.$
17. $\int (1+\cos(4t))^{\frac{5}{2}}dt.$
18. $\int \left(\tan^2(6x)+1\right)^{\frac{3}{2}}dx.$
19. $\int \left(\cot^2(3x)+1\right)^{\frac{3}{2}}dx.$
20. $\int \frac{1}{1+\tan^2(x)}dx.$
21. $\int \left(\frac{\cot^2\left(\frac{x}{2}\right)-1}{\cot^2\left(\frac{x}{2}\right)+1}\right)^{-3}dx.$
22. $\int \cos(2t)\cos(3t)dt.$
23. $\int \frac{1}{1+cos(u)}du.$
24. $\int (\csc(\theta)+\cot(\theta))^{-1}d\theta.$
25. $\int \frac{1}{1+\sec(x)}dx.$
26. $\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{1}{1-\sec(\theta)}d\theta.$
27. $\int_0^{\frac{\pi}{4}} \frac{1}{1-\sin(t)}dt.$
28. $\int_0^{\frac{\pi}{2}} \frac{\sin(2x)}{1+\cos(x)}dx.$
29. $\int \frac{\sec^3(t)}{\tan^2(t)}dt.$
30. $\int \frac{\cos^2(\theta)}{(1-\sin(\theta))^2}dx.$

31. $\int \frac{1+\sin(x)}{1-\sin(x)}dx$.
32. $\int \frac{\sin(x)}{2-\sin^2(x)}dx$.
33. $\int \frac{1-\tan^2(x)}{\sec^2(x)}dx$.
34. $\int \frac{1}{2-\sin^2(x)}dx$.
35. $\int(1+\cos(\theta))^{\frac{1}{2}}d\theta$.
36. $\int(1+\sin(2x))^{\frac{1}{2}}d\theta$.

b) Si n es un número entero impar positivo, demuestre que $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^n(x)dx = 0$.

c) Si n es un número entero distinto de cero, demuestre que $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^n(x)dx = \frac{\pi}{2}$.

d) Si $n \neq 0$, demuestre que $\int \csc^n(x)\cot(x)dx = -\frac{1}{n}\csc^{n-1}(x) + c$.

Sustitución trigonométrica

Calcule la integral que se indica:

1. $\int \frac{\sqrt{x^2-1}}{x}dx$.
2. $\int \frac{1}{x^2\sqrt{1-x^2}}dx$.
3. $\int \frac{x^2}{\sqrt{x^2-1}}dx$.
4. $\int \frac{x^2}{(1-2x^2)^{\frac{3}{2}}}dx$.
5. $\int \frac{dx}{(4x^2-25)^{\frac{3}{2}}}$.
6. $\int x^3\sqrt{4-9x^2}dx$.
7. $\int \frac{1}{(1+x^2)^2}dx$.
8. $\int(x^2+3)^{\frac{3}{2}}dx$.
9. $\int \frac{1}{u^3\sqrt{u^2-1}}dx$.
10. $\int csch(x)dx$.
11. $\int_0^1 \frac{x}{\sqrt{1+x^2}}dx$.
12. $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{(1-\theta^2)^{\frac{1}{2}}}d\theta$.
13. $\int_{\frac{1}{4}}^{\frac{3}{4}} \frac{x}{2x^4-2x^2+1}dx$.
14. $\int_1^2 \frac{1}{\sqrt{x^2+x+1}}dx$.
15. $\int \frac{1}{x\sqrt{\ln^2\left(\frac{1}{x}\right)+4}}dx$.
16. $\int \frac{x}{\sqrt{x-x^2}}dx$.
17. $\int \frac{(t-1)}{(t^2+2t-1)^2}dt$.