

Integrales impropias

a. Calcule la integral que se indica, y diga si es convergente, divergente o nada de eso.

$$(1) \int_0^1 x^{-\frac{3}{2}} dx.$$

$$(2) \int_0^{\frac{\pi}{2}} \sin(x) \cos^{\frac{1}{2}}(x) dx.$$

$$(3) \int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^{\frac{3}{2}}}.$$

$$(4) \int_{-\infty}^0 \frac{dx}{(1-x)^2}.$$

$$(5) \int_{-\infty}^{\infty} \frac{dx}{x^2+2}.$$

$$(6) \int_1^{\infty} \frac{dx}{x}.$$

$$(7) \int_{-\infty}^{\infty} \tanh(x).$$

$$(8) \int_{-\infty}^{\infty} \frac{x dx}{(x^2+1)^{\frac{3}{2}}}.$$

$$(9) \int_e^{\infty} \frac{dx}{x \ln(x)}.$$

$$(10) \int_{-\infty}^{-e} \frac{dx}{x \ln(-x)}.$$

$$(11) \int_{-\infty}^0 \sinh(x) dx.$$

$$(12) \int_0^{\infty} \sinh(x) dx.$$

$$(13) \int_0^{\infty} \frac{dx}{x+1}.$$

$$(14) \int_0^{\infty} \arctan(x) dx.$$

$$(15) \int_0^{\infty} \cot^{-1}(x) dx.$$

$$(16) \int_{-\infty}^{\infty} \operatorname{arccot}(x) dx.$$

$$(17) \int_0^1 \frac{dx}{\sqrt{1-x^2}}.$$

$$(18) \int_{-2}^{-1} \frac{dx}{x\sqrt{x^2-1}}.$$

$$(19) \int_2^3 \frac{dx}{\sqrt[3]{x-2}}.$$

$$(20) \int_0^{\frac{\pi}{2}} \csc(x) dx.$$

$$(21) \int_0^1 \operatorname{coth}(x) dx.$$

$$(22) \int_0^{\sqrt[3]{2}} \frac{x^2 dx}{\sqrt{1-x^2}}.$$

$$(23) \int_{-1}^0 \frac{x^2 dx}{\sqrt{x^3+1}}.$$

$$(24) \int_0^{\frac{\pi}{2}} \frac{d\theta}{1-\cos(\theta)}.$$

b. Demuestre que $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{n}{i^2} = \infty$.

c. Demuestre que $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n(1-\sin(\frac{i}{n}))} = \infty$.